BY MIKE WOOD

MacAdam ellipses

Due to the increasing use of solid state lighting devices, utilizing MacAdam ellipses to define the color point specification of lamps is becoming more prevalent

I'VE TALKED ABOUT MACADAM ELLIPSES in these articles more than once, and given brief descriptions about what they are and their relevance to our industry. However it's still a topic that's not as well or widely understood as it should be, so I believe it bears a longer exposition. MacAdam ellipses have long been used to define the color point specification of lamps, and that use is becoming even more prevalent as we move to using solid state lighting devices where the necessary evil of binning becomes an essential concept.

MacAdam ellipses provide a guideline as to how accurate the average person's color vision is, and how good they are at distinguishing between similar colors. MacAdam ellipses, like just about all photometric topics, are not absolute measurements based on some definable physical property such as weight, length or time; instead, they are derived from statistical measurements of the vision properties of samples of the population. There are no absolutes in vision science—just comparative statistics. All we can measure is how the value of a particular parameter compares to what the average person sees.

Clearly our color vision has limits; we know that there are colors that are so close together that we cannot distinguish between them. We also know from personal experience that our ability to discriminate between colors varies depending on how good the lighting is, how saturated the colors are, and, to some extent, which particular hue we are talking about. Note that I'm not talking about color blindness here, except that we are all color-blind to some extent, but rather about the limitations of the so called normal observer.

MacAdam ellipses are a relatively recent concept. David MacAdam was a color scientist working with Kodak's research laboratory who, in the forties, conducted a series of tests to establish the ability of test subjects to discriminate between colors. He set up experiments where the subject had to match one color against another by adjusting something very similar to the RGB or CMY controls of an automated light, and then measured how good a job they did. By getting them to attempt matching from different directions—for example, match a yellow starting with red or match the same yellow starting with green—he was able to establish zones around the target color where the user saw a range of hues as identical. He asked each subject to do this for 25 different colors across a range of hues and saturations. These zones, when plotted on the 1931 CIE xy chromaticity color space, appear approximately as ellipses. After carrying out a number of these tests, MacAdam established that there was statistically significant similarity between the results from different subjects, and he was able to formulate an average set of the ellipses that now bear his name. *Note: MacAdam didn't actually use that many subjects in his tests, so his original data is arguably statistically weak. However subsequent repeats of the experiment by other researchers have confirmed the data, and it is now widely accepted, or at least widely used!*

Figure 1 shows the standard set of 25 MacAdam ellipses. Each ellipse contains colors that cannot be differentiated by the average observer when they are at the same luminance. MacAdam estimated that someone with normal trichromatic color vision can distinguish about seventeen thousand unique colors at each level of luminance, or about three million perceivable colors overall. (Which means that correctly scaled 8 bit RGB color control plus an intensity channel should be more than enough to define every possible static color, even if it isn't enough to produce glitchless cross fades!)

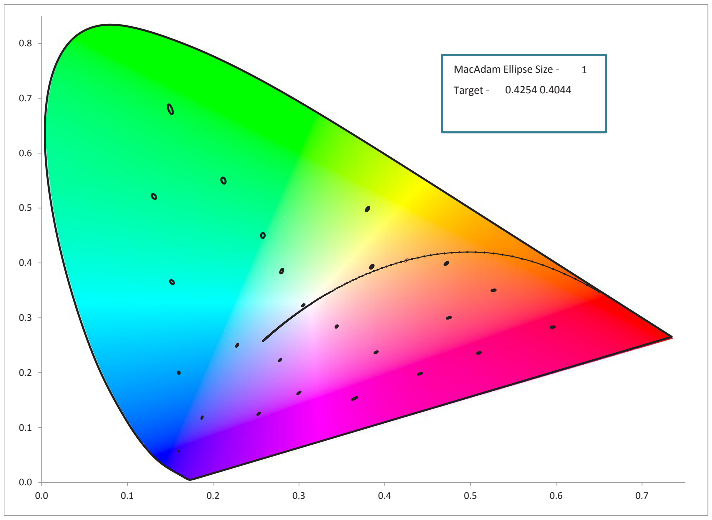

Figure 1 – 1-step MacAdam Ellipses plotted on the 1931 CIE color space

The original ellipses in **Figure 1** are very small so you don't usually see them published this way. They are often called 1-step ellipses where the 1-step means that they encompass 1-step of color discrimination around the center point. (Another way of defining a 1-step MacAdam ellipse is that it is the region on a chromaticity diagram that contains all the colors that are indistinguishable to the average human eye from the color at the center of the ellipse.) Instead, most of the time you'll see MacAdam ellipses scaled up to a larger size, perhaps 7x or 10x the original. **Figure 2** shows the same ellipses but now scaled up to 7-step ellipses. This means that each ellipse now represents seven times the just noticeable difference in color at its center point. That means that two points at the extremes of the ellipse are actually 14 steps away from each other.

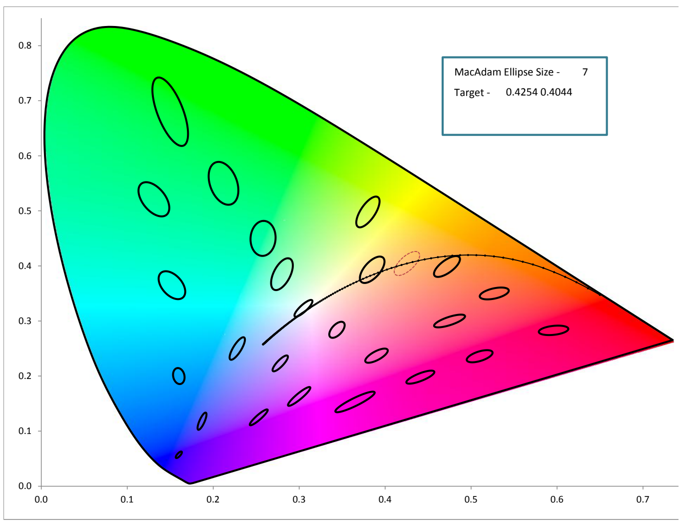

Figure 2 – 7-step MacAdam Ellipses plotted on the 1931 CIE color space

You can now start to see some detail and differences between the ellipses. For example, when plotted on the 1931 chromaticity space like this, we see that the ellipses in the blue region are much smaller than those in the green. To some extent this shows that we are more discriminating in blue hues than green, but, although this is somewhat true, this almost is as much a result of the color space we've chosen to plot the results on as it is of the human eye response. However, the 1931 color space is the one everyone is most familiar with, so it makes sense to use it. I've also shown the Planckian locus or black body line on the diagram, and you can see that the ellipses in that area tend to be very roughly aligned such that their long axis is parallel to that locus. That means that we are less sensitive to color differences along the black body line, which equate to red/blue shift or differences in color temperature, than we are to differences across it, which equate to the green/magenta shift. I think you'll agree that's true: we tolerate quite significant amounts of change in color temperature without concern whereas even a small amount of shift to the green or magenta is noticeable.

You might also notice that I've plotted an extra, 26th, ellipse as a red dotted line. This is centered on 3200 K on the black body line and thus represents the very familiar color point of theatrical incandescent lamps. (The creation of new ellipses like this isn't entirely proper. The data is statistical and specific, not mathematical, and doesn't necessarily lend itself to extrapolation and interpolation. However the data is consistent and smooth enough that such mathematical constructs seem to behave well. In this case I created the new ellipse at 3200 K by extrapolating from the full set of 25 known ellipses using a 4th power weighting based on their x,y planar distance. **Figures 1** and **Figure 2** come from the Excel sheet I created to allow the synthesis of a MacAdam ellipse anywhere on the x,y chart. I don't know if this is a standard technique but it works for me!) Let's zoom in on that ellipse so we can take a look in greater detail. **Figure 3** shows a close up of the 3200 K point with the black body line and three sizes of MacAdam ellipses, 2-step, 4-step, and 7-step.

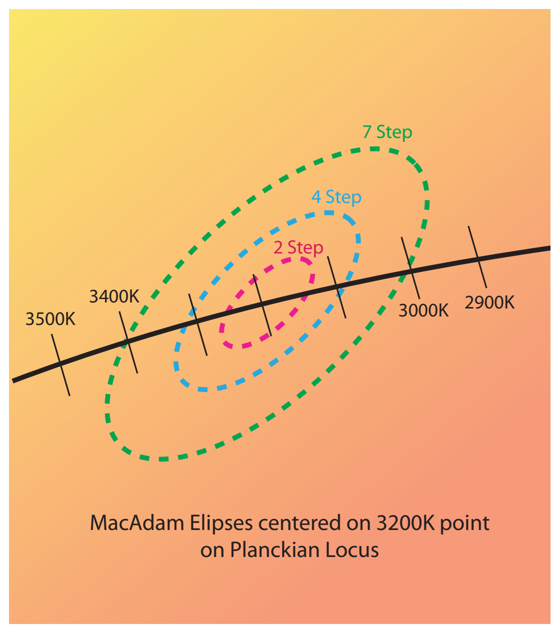

Figure 3 – MacAdam Ellipses at 3200K point

Why did I choose these three sizes of ellipse? Well, there's method in my madness: these are the three sizes that you'll often see used in the color specifications for light sources. In particular, the 7-step is the standard for color consistency in CFL lamps and in ANSI/NEMA/ANSLG C78.377-2008 American National Standard for Specifications for the Chromaticity of Solid State Lighting (SSL) Products (which is a standard for white light, not colored).

Looking at **Figure 3** you can see that a 7-step ellipse centered on 3200 K covers a very broad area and spans a 500 K range of color temperatures from 2950 K to 3450 K. The 4-step and 2-step ellipses are tighter standards, which have been proposed by the Lighting Research Center of the Rensselaer Polytechnic Institute for use in LED luminaires, and are also standards that many light source manufacturers are already working to. The 4-step ellipse is also the standard to which many entertainment industry HID lamps, such as MSR and HMI, have been manufactured for many years. At the other extreme, we are used to working with incandescent lamps, which are extremely consistent and fall well within a 1-step ellipse. Realistically, as we look more and more at LEDs as viable light sources for entertainment use, I believe that in most cases in our industry we should strive towards the 2-step ellipse as being a good compromise that is accurate enough most of the time.

Figures 4, 5 and **6** attempt to illustrate in more detail what these color differences might look like in a real situation. In each diagram the left side of the image shows three square amber color patches, one above the other. The center one represents 3200 K light, while above and below it are patches that represent the extremes of the corresponding MacAdam ellipse. On the right of each image are three more patches showing the same light sources, but this time the 3200 K yellow has been shifted to a pure white to illustrate what they might look like after the eye has fully white adapted to the color temperature. (We don't see 3200 K light as yellow light on a stage. We see it as white because it has become the reference point to our eyes.)

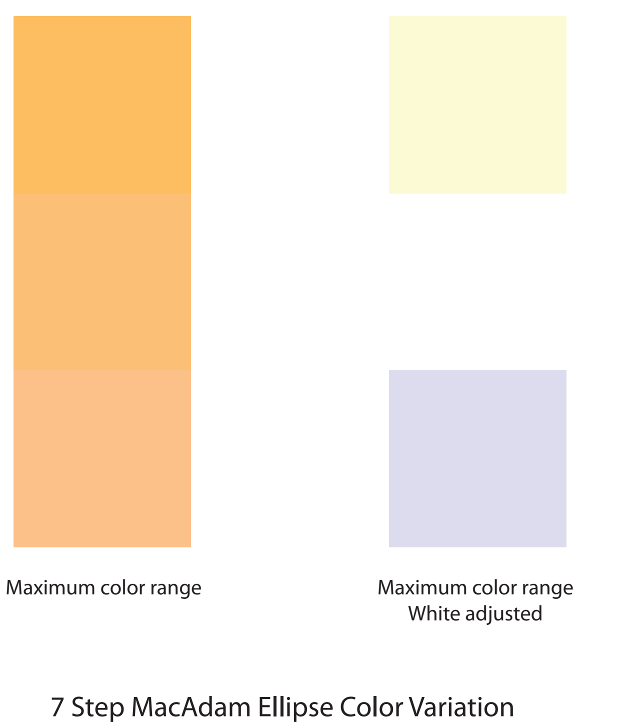

Figure 4 – White color variation at 3200 K: 7-step

Maximum color range

Maximum color range
White adjusted

4 Step MacAdam Ellipse Color Variation

Figure 5 – White color variation at 3200 K: 4-step

Maximum color range

Maximum color range
White adjusted

2 Step MacAdam Ellipse Color Variation

Figure 6 – White color variation at 3200K: 2-step

> “. . . we should strive towards the 2-step ellipse . . .”

I don't know how well the printing process used to produce this journal will render these color patches, but I hope that it's good enough to at least give you an idea of the subtlety of the situation and how even a 2-step ellipse may be visible in some circumstances. While our main light sources have primarily been incandescent lamp-based we haven't had to worry too much about their color consistency. We knew that discharge lamps varied quite a lot from batch to batch and over their life but, for many people, it wasn't of huge concern. However, as we move into the realms of SSL and introduce further variables, such as the vagaries of LED binning, knowledge of how closely the eye can discriminate between colors and how you can measure and specify that discrimination will become increasingly important to everyone. ■

Mike Wood is President of Mike Wood Consulting LLC which provides consulting support to companies within the entertainment industry on technology strategy, R&D, standards, and Intellectual Property. A 30-year veteran of the entertainment technology industry, Mike is the Treasurer and Immediate Past President of ESTA. Mike can be reached at mike@mikewoodconsulting.com.